LFG SYSTEM DESIGN UTILIZING THE FINITE ELEMENT METHOD

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ABSTRACT

Landfill gas flow is governed by numerous chemical, physical and biological processes. The migration of landfill gas (LFG) can cause problems such as asphyxiation, development of explosive conditions, groundwater contamination and odors and health risks. A common goal of most landfill gas control systems is to mitigate these and other concerns associated with landfill gas. This paper presents finite element modeling techniques that can be used by a LFG system designer to help optimize LFG collection systems. Two dimensional steady-state and transient finite element models are presented as examples.

BACKGROUND

Finite element analysis (FEA) was first presented in a paper by Turner, Martin, Clough and Topp (1956). The first uses of FEA were for structural analysis in the aeronautical industry. Since that time, the finite element method has been used in all fields of engineering including structural analysis, thermal analysis, electromagnetic analysis and fluid analysis. The processes affecting the migration of landfill gas are similar to those affecting groundwater migration and solute-transport analysis. Many of the methodologies used in groundwater analysis can therefore be applied to landfill gas.

Landfill gas migration is governed by many complex physical and chemical processes. The primary process which moves landfill gas is advection. Advection is described by the Darcy equation and is caused by a pressure gradient usually resulting from the production of landfill gas. Landfill gas also migrates due to mechanical dispersion. As the LFG migrates from the landfill due to a pressure gradient, there will be small changes in velocity as the LFG passes through the porous media. Thus as landfill gas migrates, it will disperse to some extent throughout the porous media. Additionally, LFG will migrate due to chemical diffusion. As described by Fick's law, the LFG will migrate based on a concentration gradient. Numerous additional, secondary processes also effect LFG migration. A few of these include barometric pressure fluctuations, the attenuation of methane in the soil due to methanotrophic bacteria, the absorption of landfill gas constituents into groundwater as governed by Henry's Law, and chemical reactions with certain types of reactive or adsorptive soils. These additional processes can also be incorporated into a finite element model.

METHODOLOGY

A specific two dimensional steady-state finite element model was developed for the purposes of this paper. The model was developed in Visual C++ and uses Galerkin's Method of weighted residuals (Istok, 1989). The model solves the two-dimensional flow equation for flow through porous media:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = q$$

TABLE 1 - VALUES OF PERMEABILITY

	K_x	**K_z**	**Equivalent Alluvial Medium**
Refuse	3.927 ft./day	0.3927 ft./day	Medium sand
Formation	0.8202 ft./day	01.64 ft./day	Fine sand
Clay cap	0.0262 ft./day	0.00524 ft./day	Silt

Where:

K_x = permeability in x-direction
K_z = permeability in z-direction
h = pressure head
q = landfill gas generation

The finite element method solves for pressures throughout the formation by breaking up the formation into discrete elements. Figure 1 shows the network of elements and nodes used for this specific landfill model. The landfill model is exploded and shows the landfill cap, the refuse, and the surrounding soil.

Only one half of the landfill is represented because symmetry is assumed. Underneath the formation, a no-flow boundary is applied to represent groundwater. At the surface of the landfill and the surrounding formation, a pressure of zero is defined at each node as a Dirichlet boundary condition. Permeabilities can be defined separately for each element in the formation. The permeability is defined as follows (Todd, 1980):

$$K = \frac{s_g}{m_g} K_s$$

Where:

K = permeability
K_s = specific permeability
σ_g = density of gas 0.07289 lb/ft^3
μ_g = absolute viscosity of gas 2.76 x 10^{-7} lb sec/ft^2

Using this equation, known soil permeabilities for water can be converted to permeabilities for gas. Permeabilities in the X and Z directions (K_x, K_z) are not equal but can vary significantly. Typical ratios of permeabilities (K_x/K_z) fall in the range of 2 to 10 for alluvium and can be up to 100 or more where clay is present (Todd, 1980). Values for permeabilities used in the model are tabulated in **Table 1** above.

Flow rates were defined for landfill gas generation. The flows in the finite element method cannot be directly applied to the elements, therefore the flow in each element was split between the surrounding nodes as Neumann boundary conditions. Methane generation in landfills is most commonly described by a first order decay equation.

The flow rate of LFG is described as follows (Emcon Associates, 1982):

$$Q = 2kLoMe^{-kt}$$

Where:

k = rate constant: 0.036 year^{-1} (20 year half-life)
Lo = methane yield = 1 ft^3/lb refuse
M = 44.44 lb/ft^3 (lbs of refuse per ft^3 of landfill)
t = time (10 year old refuse was used in the model)
Q = 0.0596 ft^3/day/ft^3 of landfill

By multiplying the flow rate by the area of each element, an individual flow rate can be applied to each node in the landfill. In this example, the refuse was assumed to be uniform, however, a differential generation rate can be applied to each node in the landfill to accommodate refuse of different ages and compositions.

Results of the modeling are given on the pressure contour maps shown on Figures 2, 3 and 4.

Figure 2 shows the pressure profile in the landfill following landfill closure. That is, the landfill has a clay cap applied. Modeling results show a reasonably constant pressure within the landfill. Because of the low permeabilities assigned to the clay cap, surface emissions are impeded and most of the gas migration is lateral through the subsurface.

Figure 3 shows the effects of removing the clay cap. The pressures in the landfill drop by more than 200%.

Figure 4 shows the effects of adding horizontal wells in a closed landfill. The wells for this case are operated passively (no vacuum). They were applied as Dirichlet boundary conditions as nodes with zero pressure. If horizontal wells were added to a theoretical landfill with low permeability cap, the pressure below the cap would be less than 0.3 inches water column.

One weakness of the steady state model is that temperature gradients throughout the formation were not incorporated. If the gradients are known, though, this information can also be incorporated into the model as the pressures and viscosities of the gas changes. The model also assumes that the gas densities and viscosities are constant throughout the formation. This is not the case because the gas constituents and pressures may vary throughout the landfill. The differential pressures are small within the landfill and the gas properties of air and landfill gas are so similar that this doesn't present a large problem.

Once the pressure gradients are established from the steady-state model, a velocity profile can be easily established. Velocity data can be incorporated into a new transient flow model which will model the movement of methane or other constituents in LFG. The model can then predict subsurface migration of landfill gas taking into account advective, dispersive and diffusive forces.

Diffusion is normally insignificant compared to the advection and dispersion of landfill gas and can often be neglected. Diffusion in a porous media is described by Fick's law for diffusion in sediments. For one dimensional flow in the x-direction, the equation is as follows:

$$F_x = -D^* \frac{dC}{dx}$$

Where F_x is the mass flux, C is the concentration and D* is the modified diffusion coefficient described as follows (Domenico, 1990):

$$D^* = \frac{n}{t} Dd$$

Where:

n = porosity
τ = tortuosity
Dd = diffusion coefficient

The advection of landfill gas in one dimension is described by the following equation:

$$F_x = V_x C$$

Where: V_x is the velocity in the x-direction

Mechanical dispersion is a spreading process caused by the tortuous flow paths which the LFG follows. For two dimensional flow:

$$F_x = -nD_{xx} \frac{\partial C}{\partial x} - nD_{xy} \frac{\partial C}{\partial z}$$

$$F_y = -nD_{yy} \frac{\partial C}{\partial y} - nD_{yx} \frac{\partial C}{\partial x}$$

Where: D_{xx}, D_{yy}, D_{xy} and D_{yx} are the coefficients of mechanical dispersion. They are described as follows:

$$D_{xx} = \left(a_T \bar{V}_y^2 + a_L \bar{V}_x^2 \right) \div \left| \bar{V} \right|$$

$$D_{xx} = \left(a_T \bar{V}_x^2 + a_L \bar{V}_y^2 \right) \div \left| \bar{V} \right|$$

$$D_{xy} = D_{yx} = \left[\left(a_L - a_T \right) \bar{V}_x \bar{V}_y \right] \div \left| \bar{V} \right|$$

Where: $\left| \bar{V} \right| = \sqrt{\bar{V}_x^2 + \bar{V}_y^2}$

$\overline{V}_x$ = pore fluid velocity in the x-direction
$\overline{V}_y$ = pore fluid velocity in the y-direction
a_T = transverse dispersivity
a_L = longitudinal dispersivity

The final transport equation is as follows:

$$\frac{\partial C}{\partial t} = D_{xx}\frac{\partial^2 C}{\partial x^2} + D_{xy}\frac{\partial^2 C}{\partial x \partial y} + D_{yx}\frac{\partial^2 C}{\partial y \partial x} + D_{yy}\frac{\partial^2 C}{\partial y^2}$$

$$-\frac{\partial}{\partial x}\left(\frac{V_x C}{n}\right) - \frac{\partial}{\partial y}\left(\frac{V_y C}{n}\right) - \frac{\partial}{\partial z}\left(\frac{V_z C}{n}\right) + \frac{D*}{n}\frac{\partial^2 C}{\partial x^2} + \frac{D*}{n}\frac{\partial^2 C}{\partial y^2}$$

Using velocity data provided by the steady-state model, this equation can be solved by the finite element method for an individual time step. Concentration data provided by the previous time step can then be fed into the next iteration. Following this procedure, the concentrations of LFG can be determined for each moment in time. For the methane migration model used for this paper, diffusion was neglected. Velocity data was input from the steady-state model. Other inputs are as follows:

n = porosity = 0.3
a_L = longitudinal dispersivity = 200 feet
a_T = transverse dispersivity = 60 feet
C_i = methane concentration at landfill = 50%

The methane concentration was introduced as a Dirichlet boundary condition at each node within the landfill. The resulting methane concentrations after 5 years are shown in Figure 5.

The movement of Vinyl Chloride from the landfill was also modeled. The model considered the effects of advection, mechanical dispersion, and diffusion. Modeling diffusion presents a problem because gas concentrations change as the location and the time changes. However, the diffusion coefficient for Vinyl Chloride in air and in landfill gas are similar. The diffusion coefficients for Vinyl Chloride in air and landfill gas were calculated to be 0.111 cm^2/sec and 0.10 cm^2/sec, respectively. Using these two numbers, an answer can be reasonably bracketed. A Vinyl Chloride concentration of 1215 $\mu g/m^3$ was defined at each node in the landfill as a Dirichlet Boundary condition. A concentration of zero was defined over the surface of the landfill. This condition is valid if the wind speed is great enough to remove Vinyl Chloride which might accumulate immediately above on the landfill surface. Again, the velocity profile was input from the steady-state model. Additional model inputs were as follows:

Dd = diffusion coefficient = 0.11 cm^2/sec
T = tortuosity = 1.4
n = porosity = 0.3
a_L = longitudinal dispersivity = 200 feet
a_T = traverse dispersivity = 60 feet
C_i = initial concentration at landfill = 1215 $\mu g/m^3$

A plot after five years of Vinyl Chloride concentrations caused by advection and mechanical dispersion is shown on Figure 6. A plot of the Vinyl Chloride concentrations caused by advection, mechanical dispersion and diffusion is shown on Figure 7. It can be seen that diffusion is the primary mechanism that causes Vinyl Chloride to move downward so that it has direct contact with groundwater.

APPLICATIONS

The finite element method has many applications in the design of landfill gas systems. Potential applications are numerous, and limited only by the creativity of the designer. This section gives several potential applications of finite element modeling along with some limitations of their applications.

One application is the design and spacing of landfill gas wells. Present practice makes large assumptions in calculating the radius of influence and performance of vertical wells. The finite element method will allow for better well placement design by taking into account refuse moisture, rainfall, intermediate cover type and thickness, cell height, and other relevant landfill properties. For this application a two-dimensional model is satisfactory for modeling horizontal wells. For vertical wells, a three-dimensional model would be required or a two-dimensional model which took advantage of radial symmetry.

A major limitation in this use is the uncertainty caused by the heterogeneous nature of landfills. This application is probably more useful in dryer landfills that behave more uniformly than in wet landfills. One project designed by Gas Control Engineering required the design of passive horizontal wells for the protection of a geomembrane cap. Finite element modeling was performed to design the proper well placement locations. Modeling indicated that horizontal wells placed about 10 feet below the landfill cap would be sufficient to adequately relieve the pressure so that the geomembrane cap would be protected. Placement in this area was also selected so that the LFG system could be converted to an active extraction system and reasonable LFG collection system performance could be expected.

Another application of the finite element method with regards to landfill gas is the design of passive and active systems for the protection of structures on the landfill surface. A vent system could be modeled before construction actually began to estimate the effectiveness of different types of passive and active systems.

Groundwater contamination by LFG has been causing substantial expense to a number of landfill owners (Prosser and Janechek, 1995). One of the problems has been demonstrating to regulators that LFG collection is often times the best method of controlling contamination. Modeling can provide a graphical representation of the potential results while providing the designer the opportunity to try various design scenarios without installing actual gas collection systems.

Finite element analysis is a powerful and flexible tool for modeling LFG migration. Heterogeneous soils and gas properties can be successfully modeled, if the conditions are accurately known. The scope and accuracy of a finite element model is limited only by the computer processing power available and the engineer's knowledge of the site.

REFERENCES

1 Turner, M.J., Clough, H.C. Martin, and L.J. Topp, 1956, "Stiffness and Deflection Analysis of Complex Structures," Journal of the Aeronautical Sciences, Vol. 23, No. 9, September.

2 Istok, Jonathon, 1989, "Groundwater Modeling by the Finite Element Method," American Geophysical Union, p 34.

3 Todd, David Keith, 1980, "Groundwater Hydrology," John Wiley and Sons, pp. 64-83.

4 Emcon Associates, 1982, "Methane Generation From Landfills", Ann Arbor Science, p. 55.

5 Domenico, Patrick A. and Schwartz, Franklin W., 1990, "Physical and Chemical Hydrogeology," John Wiley and Sons, p. 368.

6 Prosser, Richard W., and Janechek, Alan H., 1995, "Landfill Gas and Groundwater Contamination," presented at the proceedings of the October American Society of Civil Engineers (ASCE) Convention.

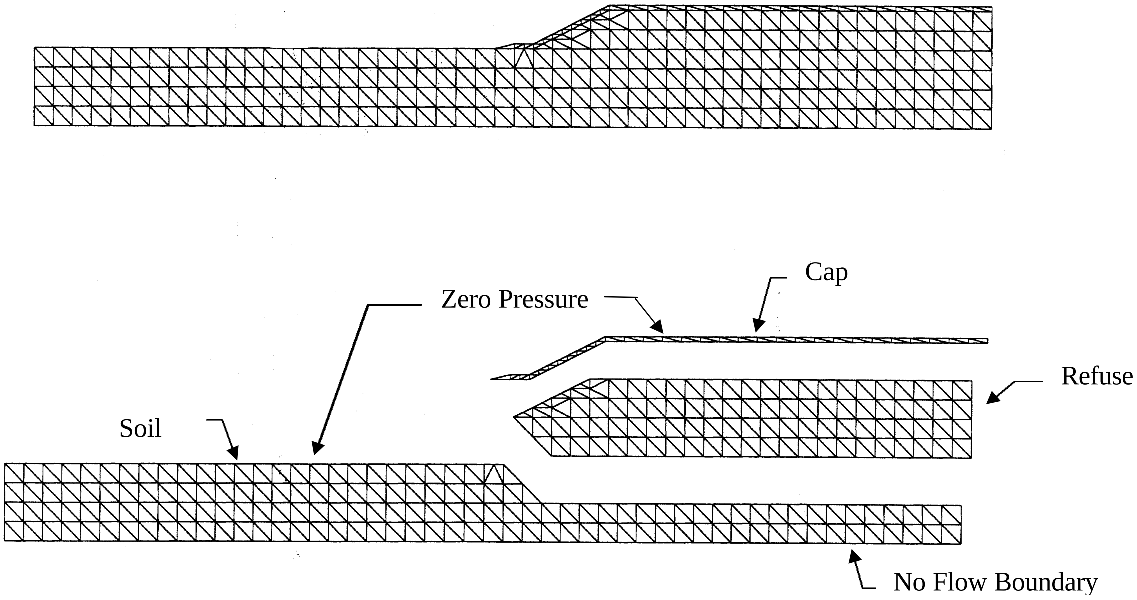

FIGURE 1 - TWO DIMENSIONAL STEADY STATE FINITE ELEMENT MODEL

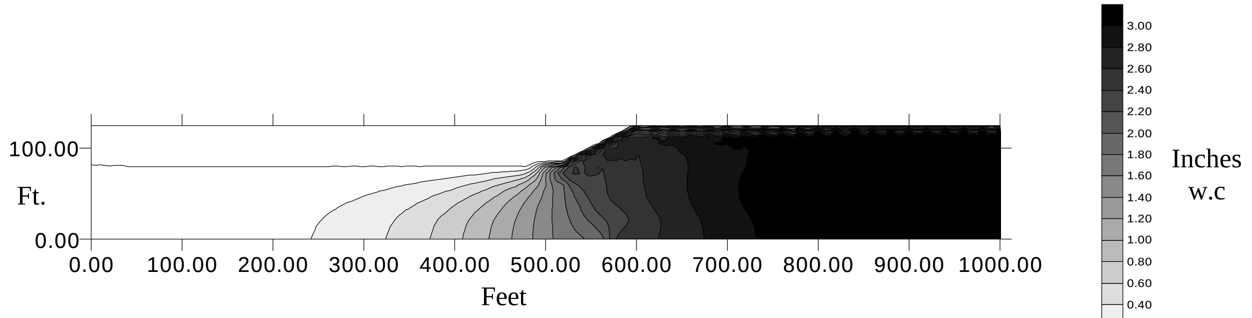

FIGURE 2 (CLOSED LANDFILL)

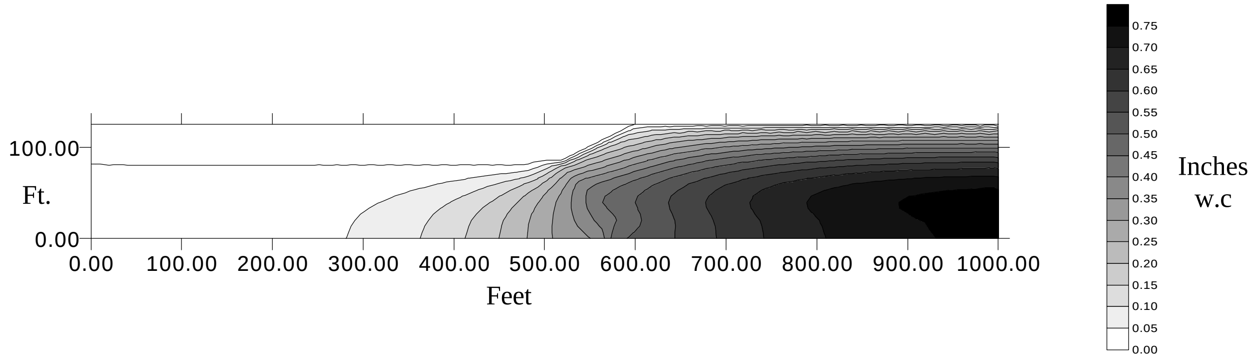

FIGURE 3 (LANDFILL WITHOUT A CLAY CAP)

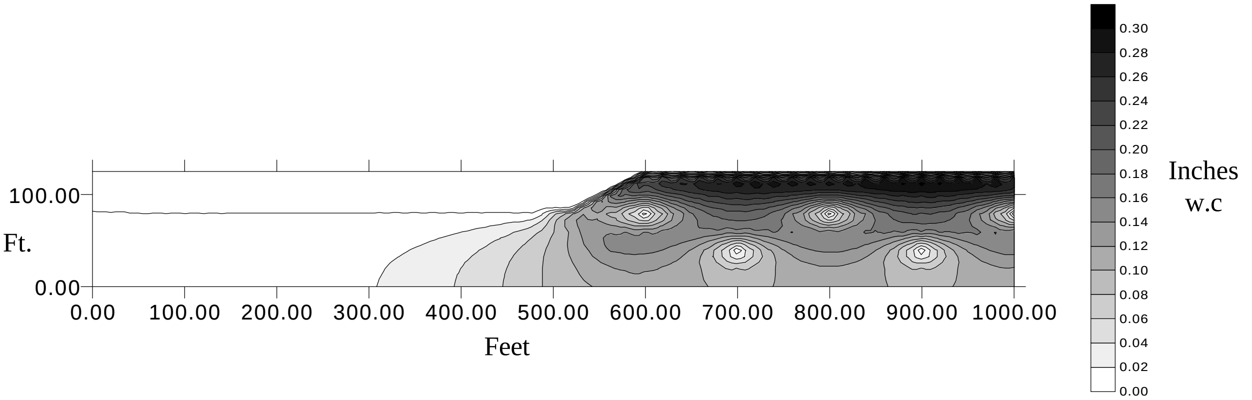

FIGURE 4 (CLOSED LANDFILL WITH 5 HORIZONTAL WELLS)

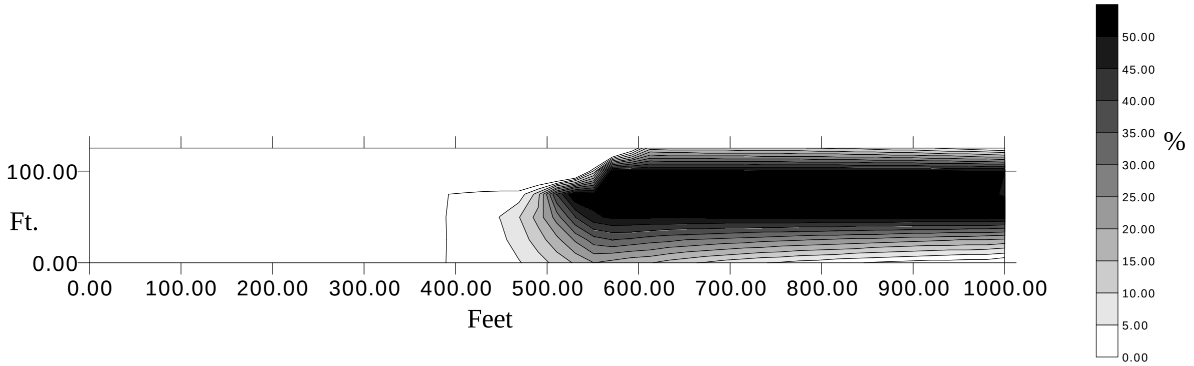

FIGURE 5

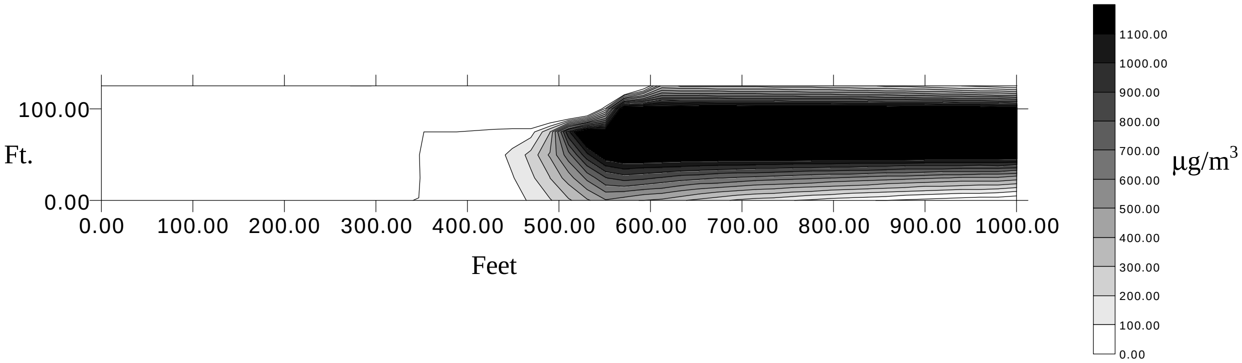

FIGURE 6

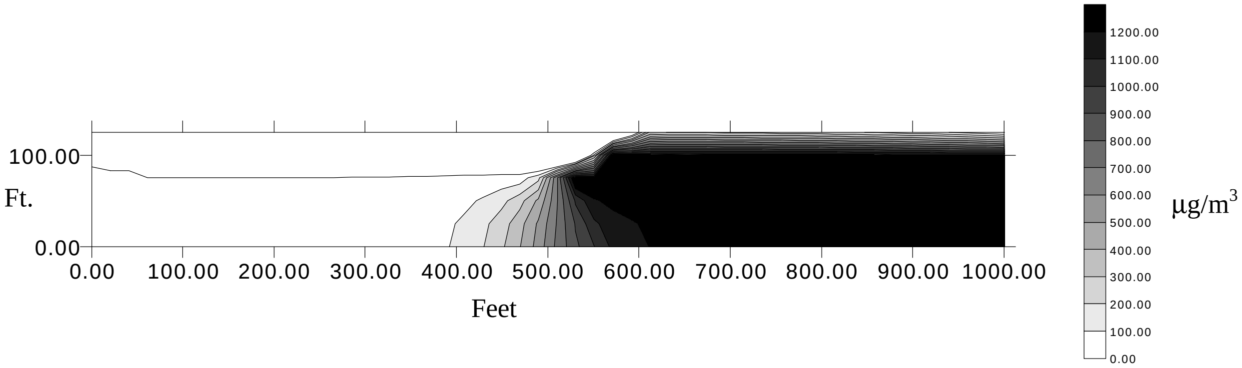

FIGURE 7